

2D GAMES AS CYBER-PHYSICAL SYSTEMS

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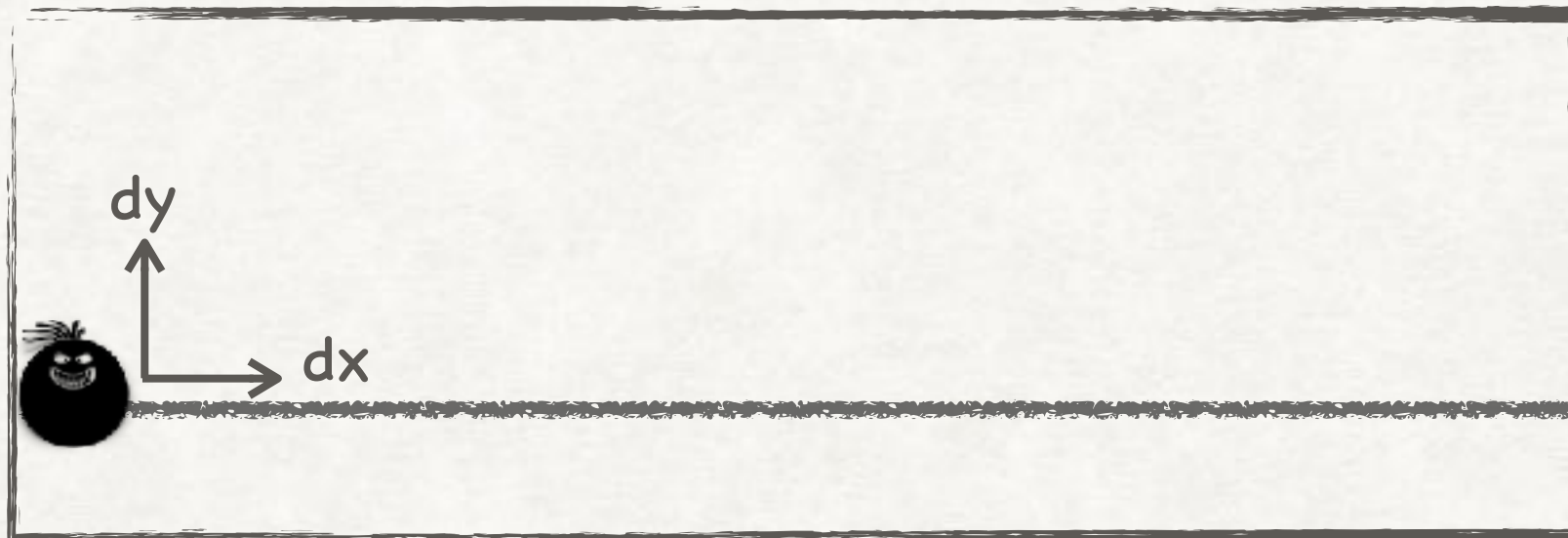
MOTIVATION

- Physics based games with discrete game controllers are hybrid systems
- Impossible games are no fun
 - Game designs need “correctness” or playability guarantees
- Trivial games are no fun
 - Games must allow “winning strategies” and be non-trivial
- Inherently adversarial — hybrid games
- Opportunity to study various elements of CPSs and dL

RELATED WORK

- Adelhart and Kargov : Mario game solver
Adelhardt, Kim, and Nedyalko Kargov. "Mario game solver." IT University of Copenhagen (2012).
- Graph analysis over axioms
- Aloupis et. al 2015 : Mario is hard
Aloupis, Greg, et al. "Classic Nintendo games are (computationally) hard." Theoretical Computer Science 586 (2015): 135-160.
- Demaine et.al 2016: Mario is easy
Demaine, Erik D., Giovanni Viglietta, and Aaron Williams. "Super Mario Bros. is harder/easier than we thought." (2016).

A (VERY) SIMPLE GAME



PLAYER DYNAMICS:

$$x' = vdx, y' = vdy$$

A (VERY) SIMPLE GAME



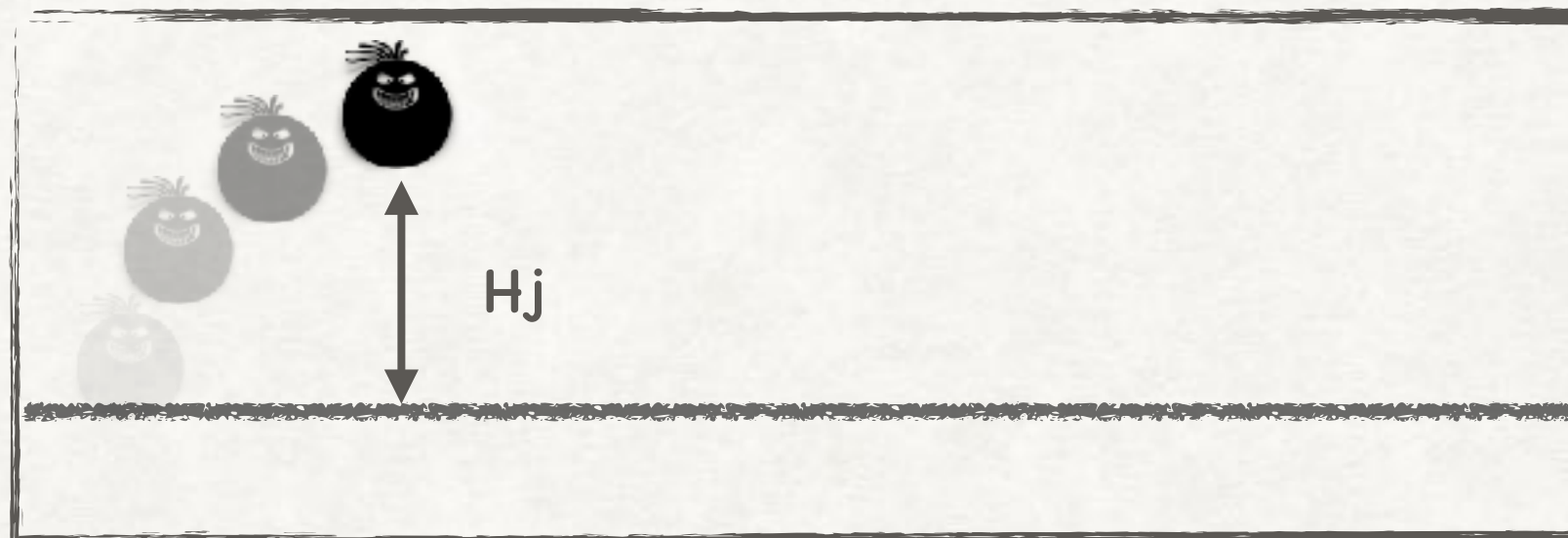
PLAYER DYNAMICS:

$$x' = vdx, y' = vdy$$

PLAYER CONTROL:

`{v:=v+1; ++ v:=v-1; ++ ?true}`

A (VERY) SIMPLE GAME



PLAYER DYNAMICS:

$$x' = vdx, y' = j + vdy, j' = -g$$

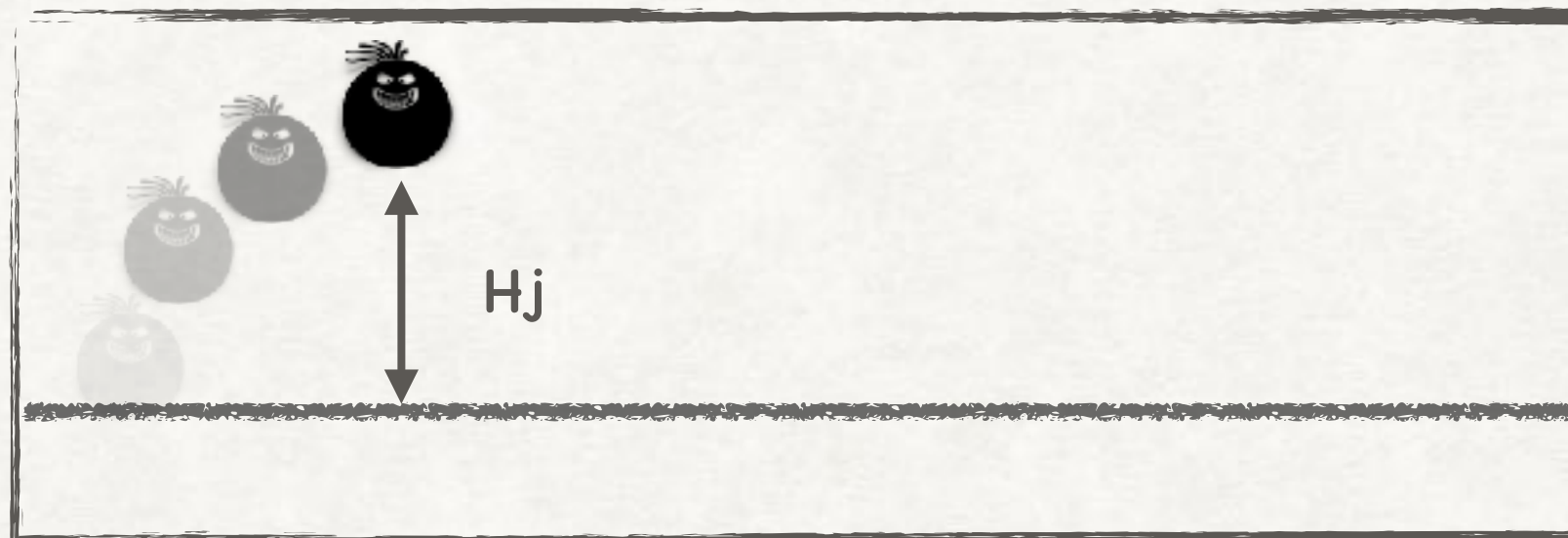
PLAYER CONTROL:

```
{v:=v+1; ++ v:=v-1; ++ ?true}  
{j:=J; g:=G} ++ {j:=0;g:=0} ++ ?true}
```

EVOLUTION CONSTRAINTS

$$y \geq \text{ground}$$

A (VERY) SIMPLE GAME



PLAYER DYNAMICS:

$$x' = vdx, y' = j + vdy, j' = -g$$

EVOLUTION CONSTRAINTS

$$y \geq \text{ground}$$

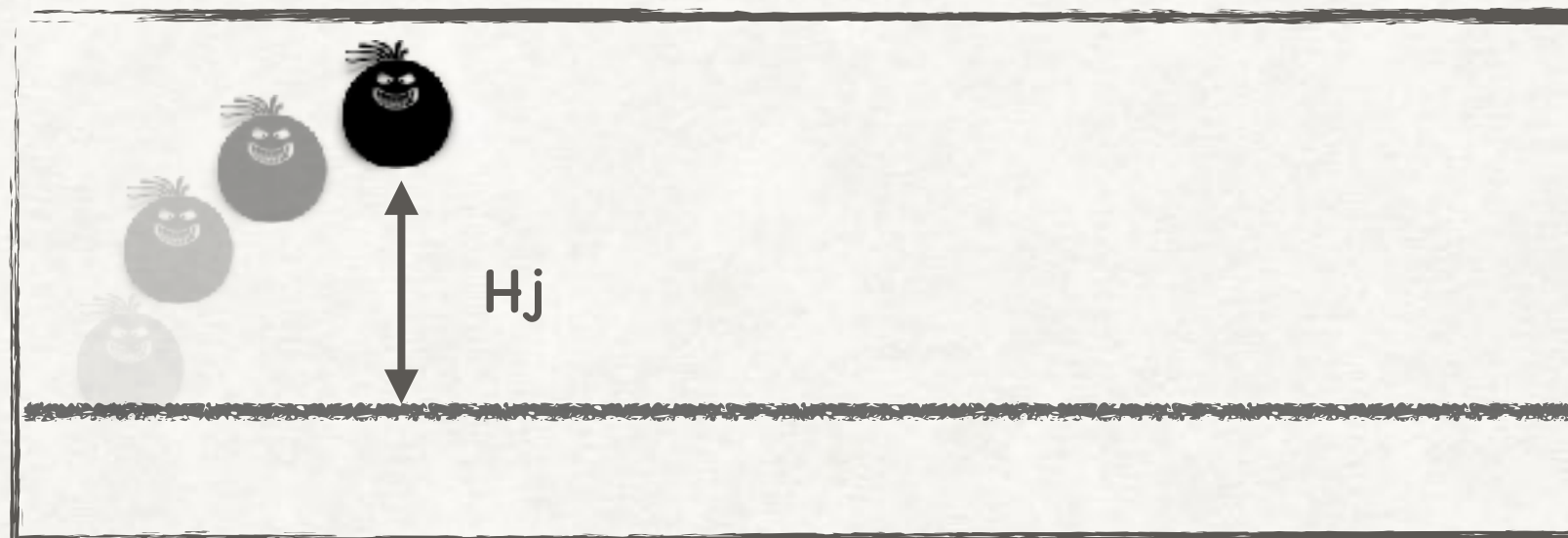
PLAYER CONTROL:

```
{v:=v+1; ++ v:=v-1; ++ ?true}  
{j:=J; g:=G} ++ {j:=0;g:=0} ++ ?true}
```

WORLD FIX-UP CONTROL:

```
if ( y <= ground ) { j:= 0; g:=0 }
```


A (VERY) SIMPLE GAME



PLAYER DYNAMICS:

$$x' = vdx, y' = j + vdy, j' = -g, t' = 1$$

EVOLUTION CONSTRAINTS

$$y \geq \text{ground} \quad t \leq T$$

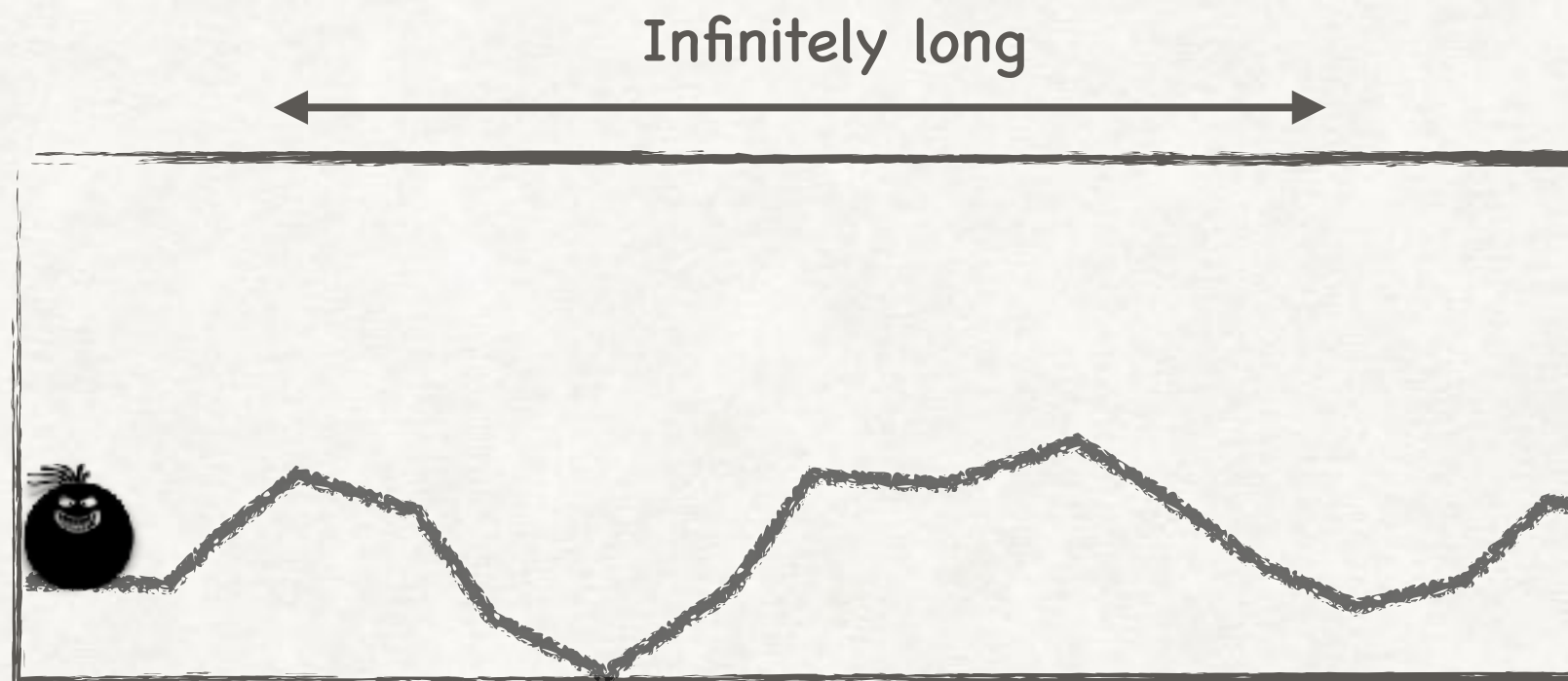
PLAYER CONTROL:

```
{v:=v+1; ++ v:=v-1; ++ ?true}  
{j:=J; g:=G} ++ {j:=0;g:=0} ++ ?true}
```

WORLD FIX-UP CONTROL:

```
if ( y <= ground ) { j:= 0; g:=0 }
```


LEVEL DESIGN



VIRTUAL PLAYER/WORLD DYNAMICS:

$$x' = vdx, l' = vdy$$

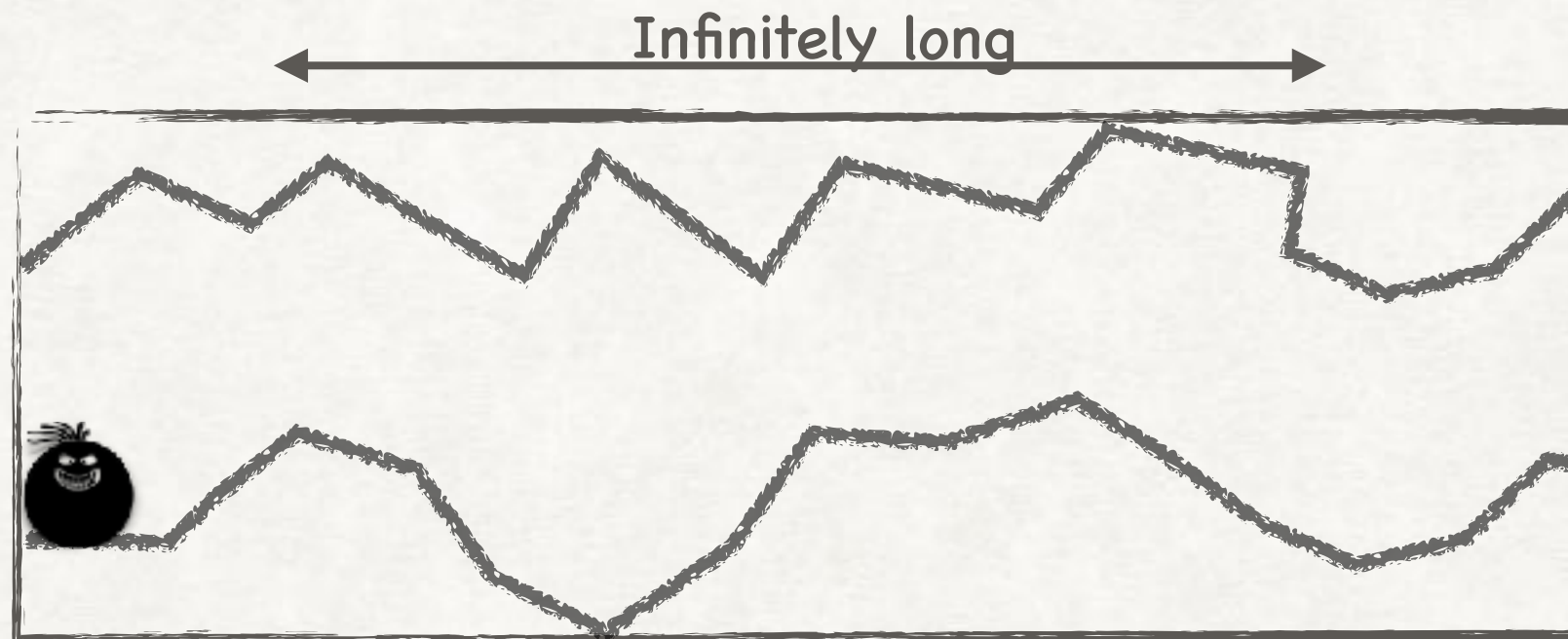
WORLD CONTROL:

$$dx:=*; dy:=*$$

$$?dx^2+dy^2=1;$$

$$? dx > 0$$

LEVEL DESIGN



VIRTUAL PLAYER/WORLD DYNAMICS:

$$ux' = v \, udx, h' = v \, udy$$

WORLD CONTROL:

$$dx:=*; dy:=*$$

$$?dx^2+dy^2=1;$$

$$? dx > 0$$

LEVEL DESIGN

PLAYER CONTROLLER

```
if( t>=T) { t:=0, ...}
```

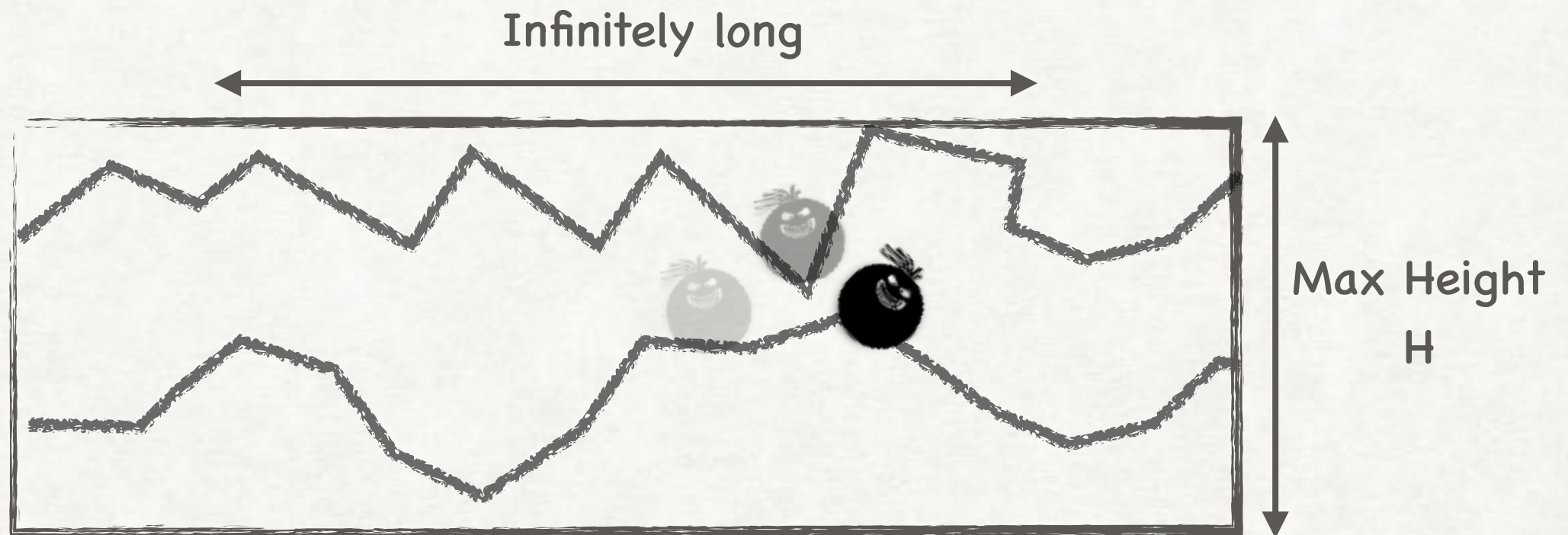
DYNAMICS

....

WORLD CONTROLLER

....

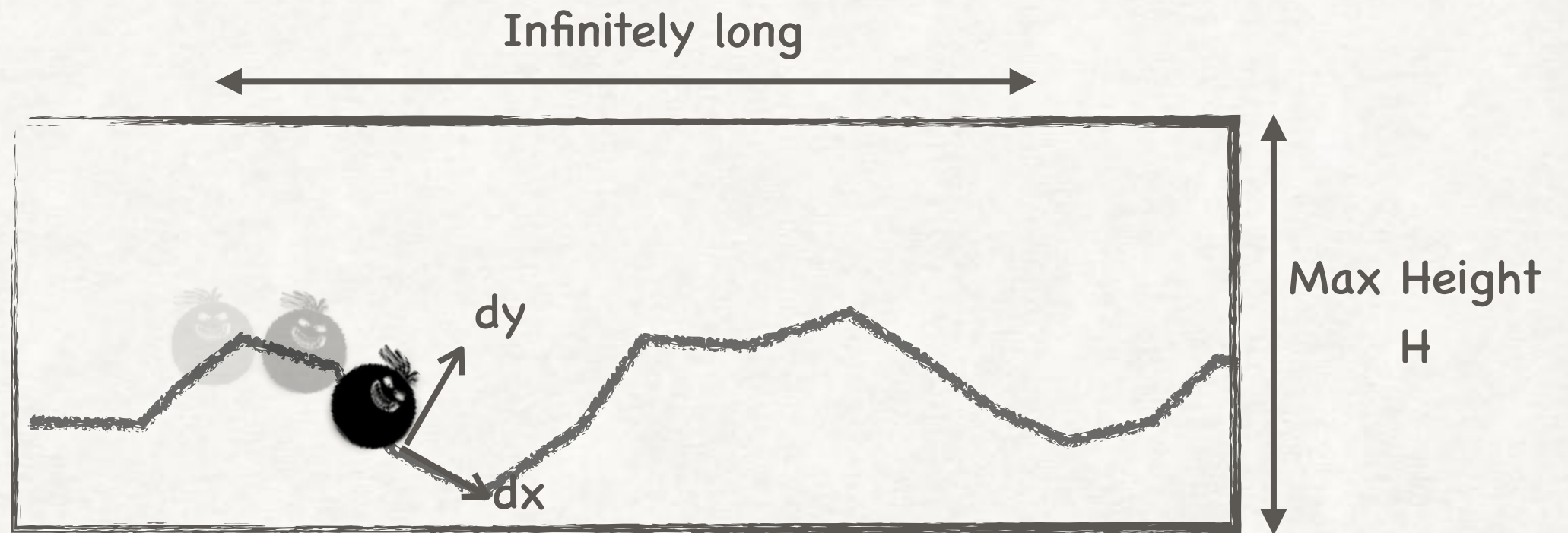
SAFETY AND PLAYABILITY



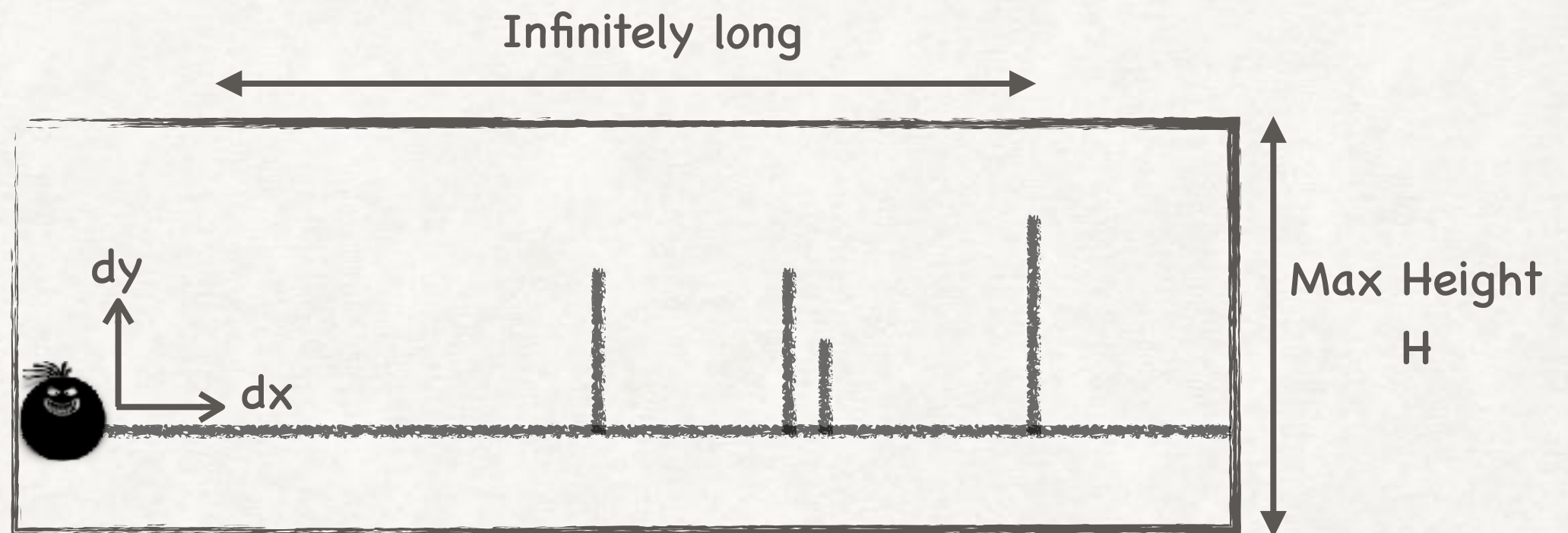
PLAYER CONTROL:
 $\text{if}(t \geq T \ \& \ y = l) \{ j.... \}$

WORLD CONTROL:
 $dx := *; dy := *$
 $?l + dy * T + \text{clearance} < H$

LEVEL MODELING

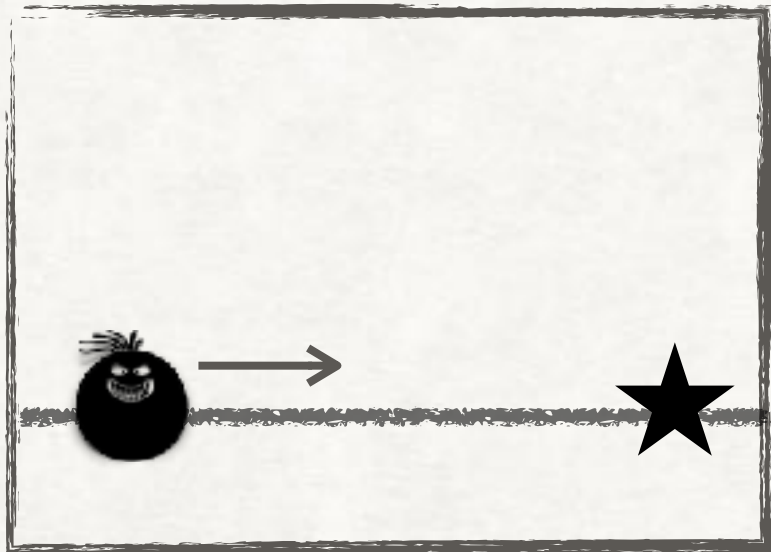


SAFETY AND *PLAYABILITY*

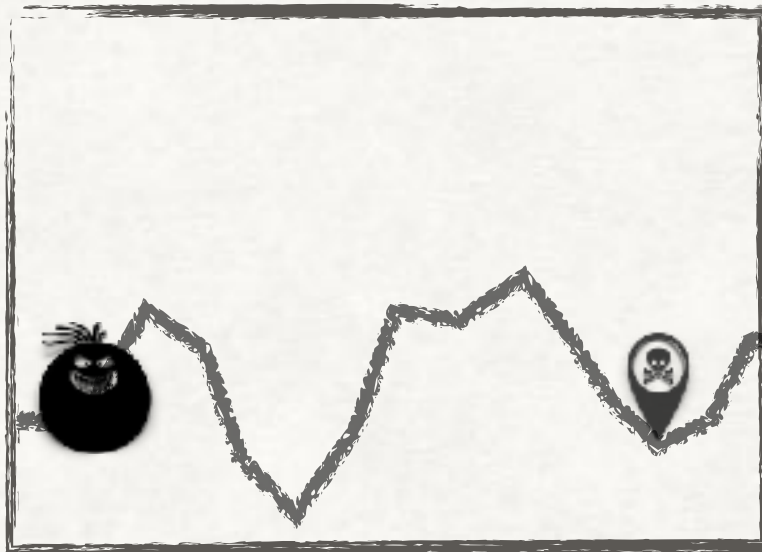


OBSTACLES:
 $hb + \text{clearance} < H$

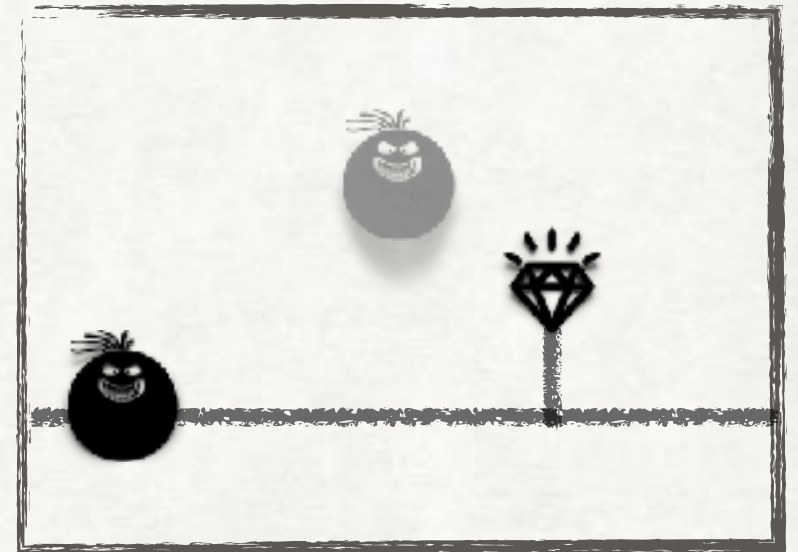
GAME CHALLENGES



PROGRESS

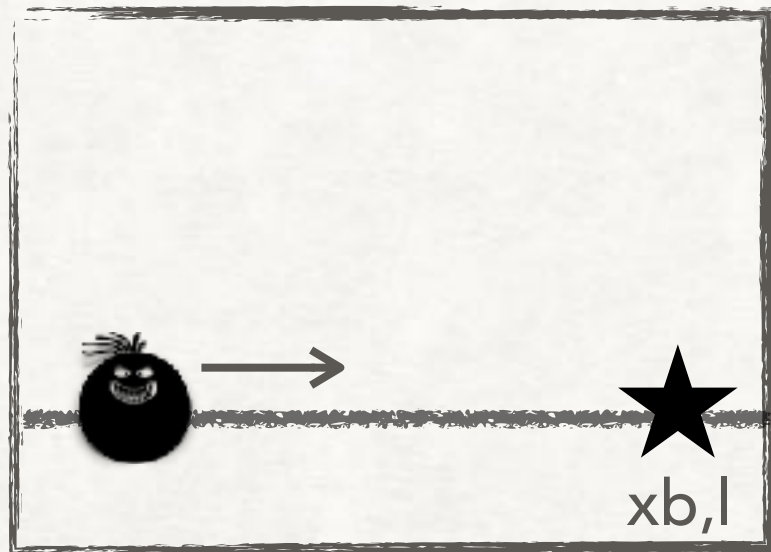


AVOID



ATTACK

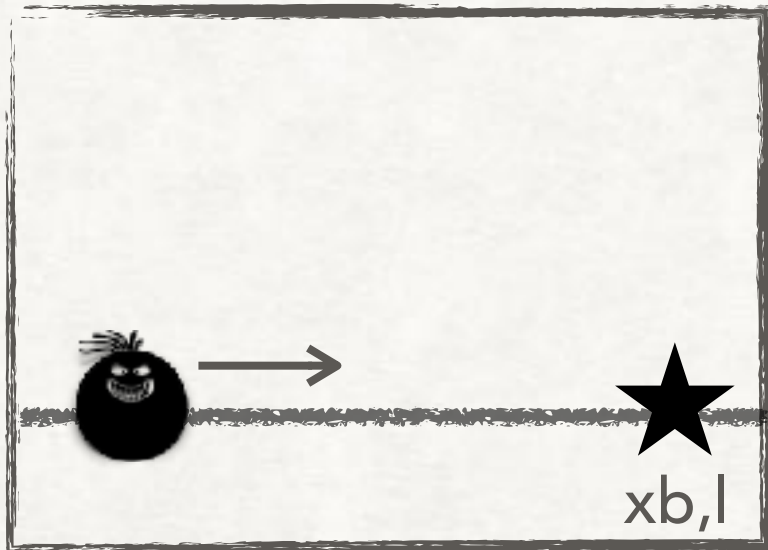
GAME CHALLENGES



PROGRESS

```
{  
  IF ( T = T )  
    T := 0  
    PLAYER CONTROLLER  
}  
  
{ ODES & T < T }  
  
{ WORLD CONTROLLER }
```

GAME CHALLENGES



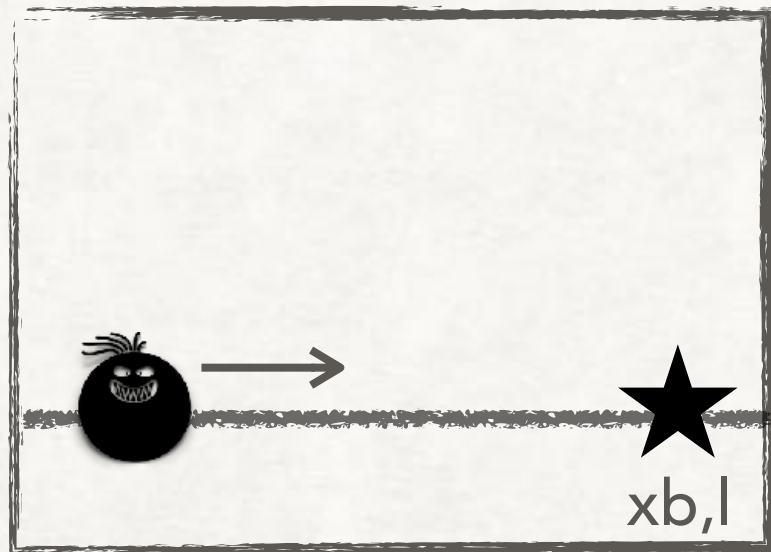
PROGRESS

```
INITIAL CONDITIONS ->
<{
  IF ( T = T)
    T := 0
    PLAYER CONTROLLER
}

{ ODES & T < T }

{ WORLD CONTROLLER}
}*
> PROGRESS?
```


GAME CHALLENGES

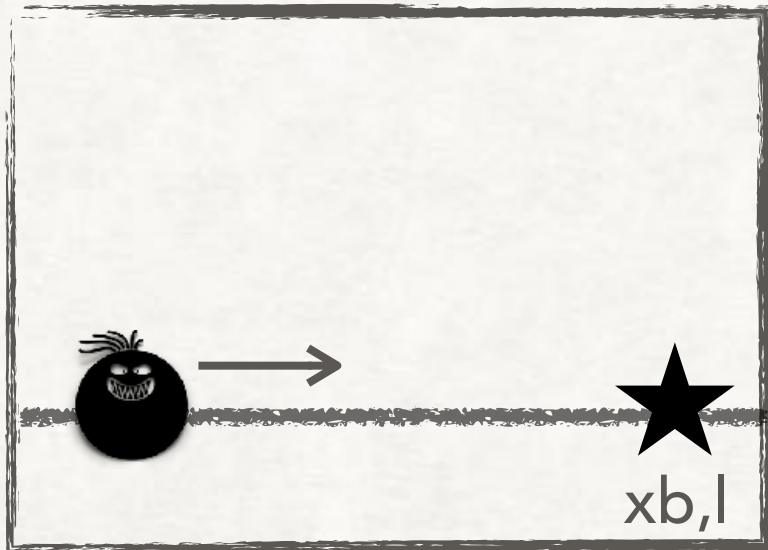


PROGRESS

```
INITIAL CONDITIONS ->  
<{  
  IF ( T = T )  
    T := 0  
    PLAYER CONTROLLER  
}  
  
{ ODES & T < T }  
  
{ WORLD CONTROLLER }  
> PROGRESS?
```



GAME CHALLENGES



PROGRESS

INITIAL CONDITIONS ->

<{

IF ($T = T$)

$T := 0$

PLAYER CONTROLLER

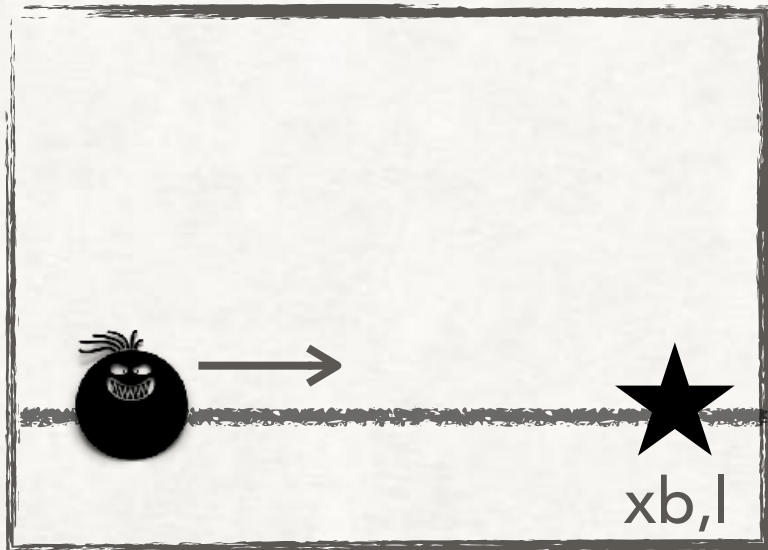
}

{ ODES & $T < T$ }

{ WORLD CONTROLLER }^@

> PROGRESS?

GAME CHALLENGES



PROGRESS

INITIAL CONDITIONS ->

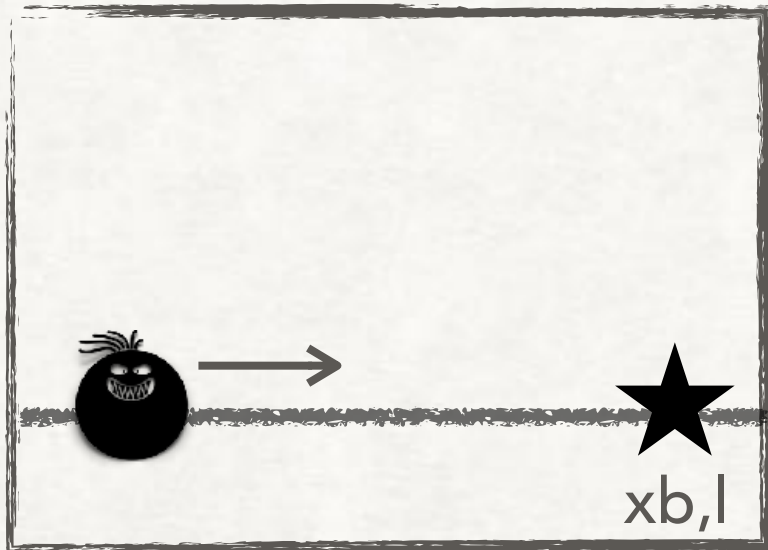
```
<{  
  IF (  $T = T$  )  
     $T := 0$   
    PLAYER CONTROLLER  
}
```

```
{ ODES &  $T < T$  }^@
```

```
{ WORLD CONTROLLER }^@  
> PROGRESS?
```

ENVIRONMENT IS AN ADVERSARY BY DESIGN

GAME CHALLENGES



PROGRESS

INITIAL CONDITIONS ->

<{

IF ($T = T$)

$T := 0$

PLAYER CONTROLLER

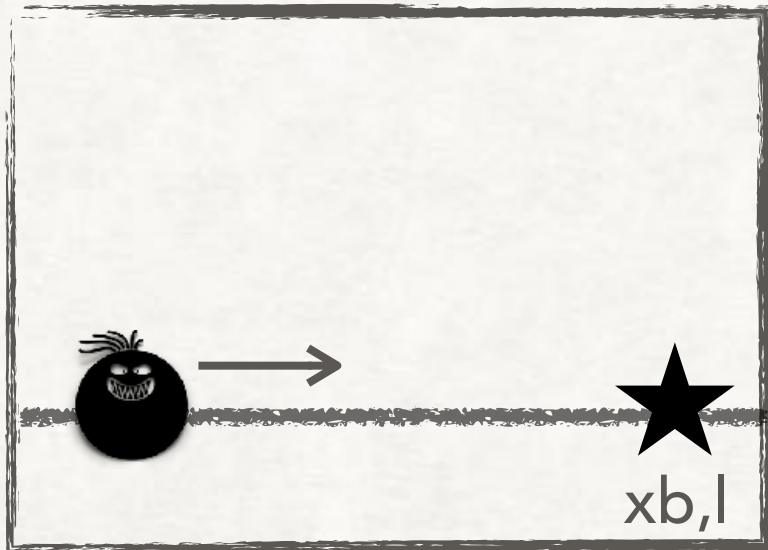
}

{ ODES & $T < T$ }^@

{ WORLD CONTROLLER }^@

> PROGRESS?

GAME CHALLENGES



PROGRESS

INITIAL CONDITIONS ->

<{

{

IF ($T = T$)

$T := 0$

PLAYER CONTROLLER

}

{ ODES & $T < T$ }^@ ?($T > 0$)^@

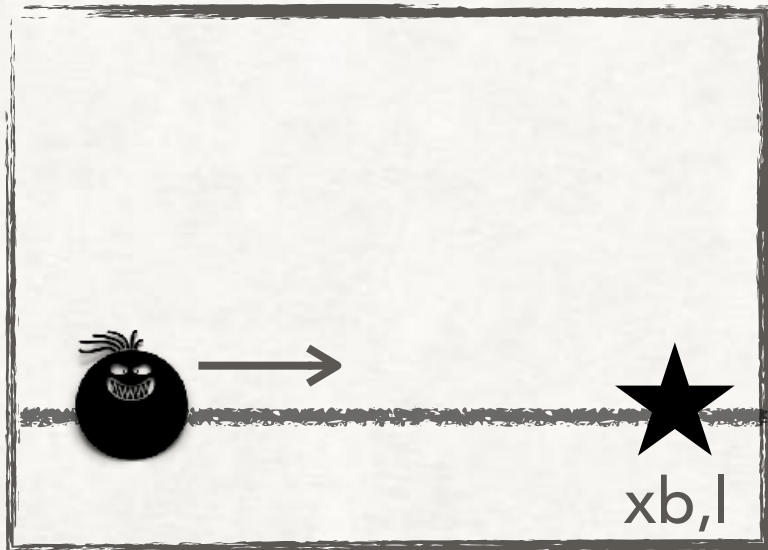
{ WORLD CONTROLLER }^@ ?($DX > 0$)^@

}*

> PROGRESS?

ENVIRONMENT IS REASONABLE

GAME CHALLENGES



PROGRESS

INITIAL CONDITIONS ->

<{

{

IF ($T = T$)

$T := 0$

PLAYER CONTROLLER

}

{ ODES & $T < T$ }^@ ?($T > EPS$)^@

{ WORLD CONTROLLER }^@ ?($DX > EPS$)^@

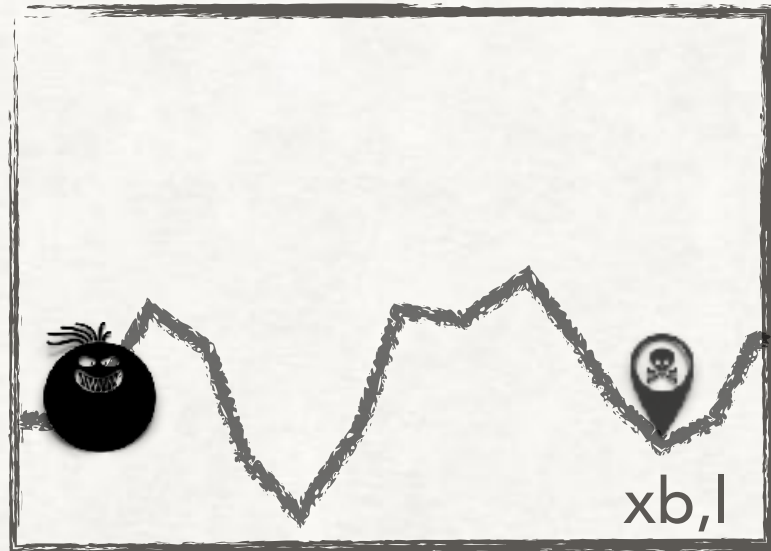
}*

> PROGRESS?

IF $V = 1$, EPS^2 PROGRESS IN EACH ITERATION

ENVIRONMENT IS "EPSILON" REASONABLE

GAME CHALLENGES



AVOID

INITIAL CONDITIONS ->

<{

{

IF ($T = T$)

$T := 0$

PLAYER CONTROLLER

}

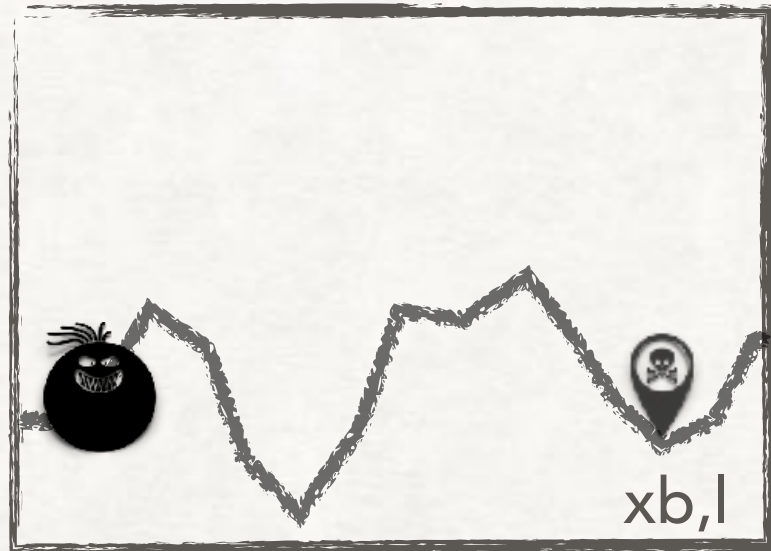
{ ODES & $T < T$ }^@ ?($T > EPS$)^@

{ WORLD CONTROLLER }^@ ?($DX > EPS$)^@

}*

> AVOID?

GAME CHALLENGES



AVOID

INITIAL CONDITIONS ->

<{

{

IF ($T = T$)

$T := 0$

PLAYER CONTROLLER

}

{ ODES & $T < T$ }^@ ?($T > EPS$)^@

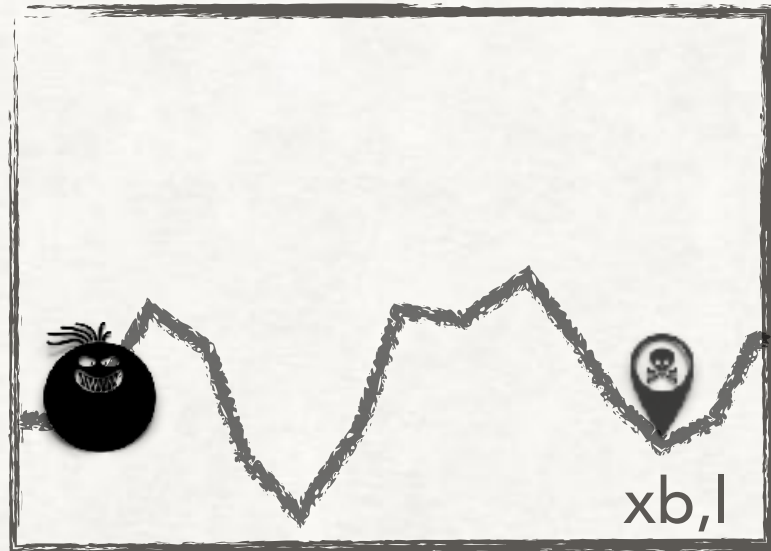
{ WORLD CONTROLLER }^@ ?($DX > EPS$)^@

}

> AVOID?

SINGLE STEP

GAME CHALLENGES



AVOID

$$TJ = (J+V)^2/2G$$

INITIAL CONDITIONS:

$$XB = X + TJ * V, T = 2 * TJ, DX > 0$$

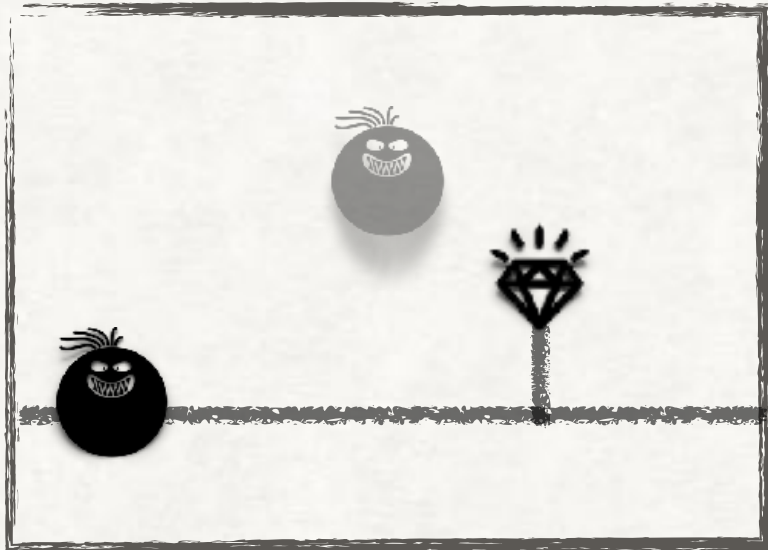
POST CONDITION:

$$X = XB \rightarrow Y > L$$

CAN MAKE "PROGRESS" ON THE LEVEL

CAN AVOID OBSTACLE BY JUMPING IF
CONDITIONS HOLD

GAME CHALLENGES



ATTACK

$$HJ = (J+V)^2/2G$$

$$TJ = (J+V)/G$$

$$TB^2 = 2(HB-HJ)/G, TB > 0$$

INITIAL CONDITIONS:

$$XB = X + (TJ + TB) * V, TJ + TB < T$$

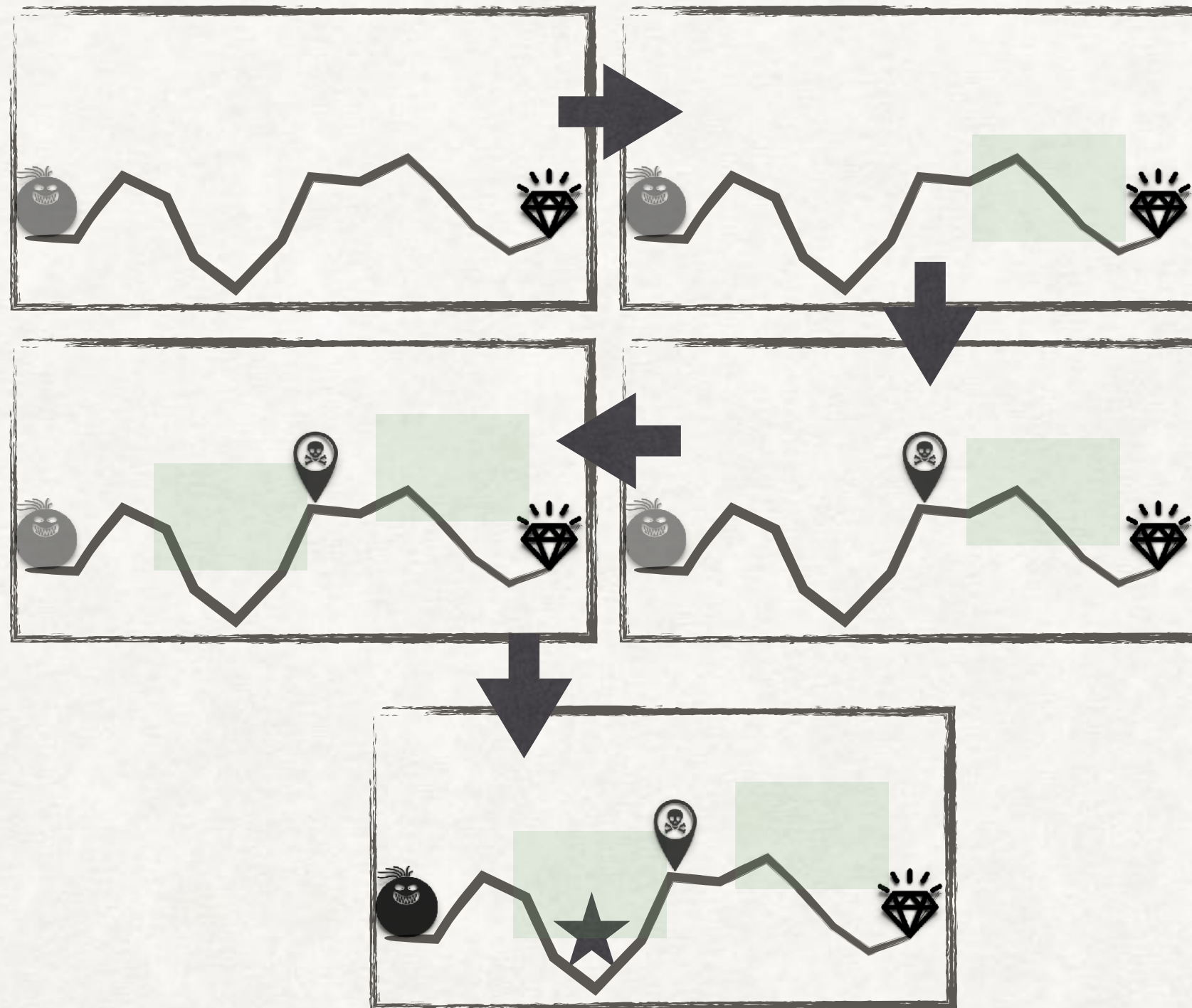
POST CONDITION:

$$X = XB \rightarrow Y = YB$$

COMPOSITION?



INTERACTIVE DESIGN?



THANK YOU!



ICON CREDITS: THE NOUN PROJECT
JOEL MCKINNEY, ICONSPHERE, CORPUS DELICTI

